An Analytical and Experimental Study for Surface Heat Flux Determination

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A numerical method by which data from a single embedded thermocouple can be used to predict the transient thermal environment for both high- and low-conductivity materials is described. The results of an investigation performed to verify the method clearly demonstrate that accurate transient surface heating conditions can be obtained from a thermocouple 1.016 cm from the surface in a low-conductivity material. Space Shuttle Orbiter thermal protection system materials having temperature- and pressure-dependent properties and typical Orbiter entry heating conditions were used to verify the accuracy of the analytical procedure. Analytically generated, as well as experimental, data were used to compare predicted and measured surface temperatures.

Nomenclature

A,B,C = quadratic coefficients
a,b,c,d = coefficients of square temperature matrix
C_p = specific heat of material at constant pressure
f = function defined by Eq. (4)
h = function defined by Eq. (3)
i = individual measurements
k = thermal conductivity of material
L = thickness of material
l = material designator
m = Beck's intermediate temperature data
P = point
$\dot{q}$ = heat flux
$\dot{q}_{conv}$ = convective heating rate
$\dot{q}_{net}$ = net heat rate
r = Beck's future temperature data
T = temperature
T' = temperature at end of time step
T_r = calculated value of temperature at node r
TC = thermocouple
t = time
x = distance or function defined by Eqs. (9) and (11)
y = function defined by Eqs. (9) and (11)
Δt = computing time, interval
Δx = node thickness
δ = convergence tolerance
ρ = density of material
Δ = denotes change in quantity

Subscripts

i = location
j,n = thermocouple location
$m,r,0,l$ = node identifier
0 = surface

Superscripts

$'$ = future time step
$*$ = known or desired value

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Introduction

THE design and development of a reusable thermal protection system (TPS) for the Space Shuttle is dependent on a detailed knowledge of the aerothermodynamic environment to which the TPS will be exposed. The TPS thermal performance normally is obtained from exhaustive plasma arc and radiant heating tests to establish reuse temperature and thermal response characteristics. In a previous study by Curry and Williams,[1] a nonlinear least-squares method was developed for the estimation of thermal property values from experimental interior temperature data. The current investigation represents the application and extension of this previously developed numerical method to the determination of surface heating rates and temperatures from measured interior temperatures.

The calculation of surface heat flux and surface temperature from an interior temperature history measurement is called the inverse heat-conduction problem and has been discussed by numerous investigators.[2-12] An excellent discussion of previous investigations[4-8] for solving the inverse problem also can be found in Ref. 2. In particular, Beck and Wolf[3] presented a method of solution using least squares and future temperatures. In a later publication, Beck[2] presented a technique using nonlinear estimation in the solution of the inverse problem. Howard[9] developed a numerical method for determining the heat flux to a thermally thick wall with variable thermal properties using a single embedded thermocouple. His best results were obtained for temperature measurements close to the heated surface in conjunction with a small computing interval.

Cornette[10,11] in analyzing the Project Fire calorimeter data, developed a transient inverse solution that required curve fitting of the basic temperature data. Cornette's solution accounted for variable material properties and yielded a closed-form analytical expression for the local surface heat flux at a given instant of time. The temperature-time data for several thermocouples embedded in a calorimeter plug were smoothed and the data replaced with a polynomial equation for temperature (at a particular thermocouple location) as a function of time. Imber and Khan[12] developed a closed-form inverse solution for constant properties and heat flux using two embedded thermocouple readings. The solution was obtained by means of Laplace transform techniques in which the input thermocouple data were approximated by a temporal power series and a second series of error functions.

The analytical method discussed in this paper, using a single embedded thermocouple, accounts for variable thermal properties (as functions of temperature and pressure) as well as for the effect of radiation losses and interior conduction. In addition, the results can be obtained with appriximately the same computational time required to solve the thermal model using a known heat rate.

The primary objectives of this paper are 1) to present a recently developed numerical method by which data from a single embedded thermocouple can be used to predict the transient thermal environment for both high- and low-conductivity materials having temperature- and pressure-dependent properties; 2) to make a direct comparison between analytically predicted and experimentally measured surface temperatures; and 3) to compare the analytical procedures described in this paper with the method of Beck.[2]

Theoretical Formulation

The problem in solving for the heating rate at a surface when the temperature at the surface is unknown is that one of the required boundary conditions is unavailable. There is no difficulty in solving for temperatures at any location between two thermocouples because both boundary conditions are known.

It is assumed that a set of thermocouples TC_j has been placed at known locations x_j in the material. This assumption implies that a set of interior temperature-time histories $T(x_j,t)$ exists. If the temperature data are available at x_j, then, by solving the heat flux boundary condition for $\dot{q}$ together with the unknown temperatures, the heat rate at location x_j can be found directly. If the thermocouple is located on the surface, the $\dot{q}_{\text{net}}$ value can be calculated directly from the tridiagonal matrix.[13]

JSC Technique

A technique has been developed at the NASA Johnson Space Center (JSC) to solve for the heat rate at the surface at each time step rather than to solve for the entire history. The technique is iterative; initially, a surface energy balance correction is used, followed by one step using the method of false position (regula falsi)[14] and then a quadratic fit. To derive the technique,‡ it is necessary first to examine the finite-difference approximation at the surface:

$$\dot{q}_{\text{net}} - \frac{T_0' - T_l'}{[\Delta x_l/(2_l k_0)] + [\Delta x_l/(2_l k_l)]} = \frac{{}_l\rho_0\, C_{p_0} \Delta x_l}{2\Delta t} \tag{1}$$

where Δt is the interval of computing time, a prime denotes the temperature value at time $t+\Delta t$, 0 and 1 are the node identifiers, and Δx is the node thickness. This form of the equation is for a composite material where l is the material designator.

Equation (1) can be rearranged into the general tridiagonal form

$$a_m T_{m+l}' + b_m T_m' + c_m T_{m-l}' + d_m = 0 \tag{2}$$

To satisfy Eq. (1) and the measured thermocouple temperature at each time step, it is desired that

$$h = T_l' - T_l'^* = 0 \tag{3}$$

where $T_l'^*$ and T_l' are the measured and predicted temperatures, respectively.

Therefore, at the surface, one can define functions

$$f = a_0 T_l' + b_0 T_0' + \frac{{}_l\rho_0\, {}_lCp_0 \Delta x_l T_0}{2\Delta t} + \dot{q}_{\text{net}} = 0 \tag{4}$$

‡This derivation assumes that the first interior finite-difference node represents the known thermocouple temperature. The extension to other interior nodes follows in a similar manner.

$$f^* = a_0 T_l'^* + b_0 T_0'^* + \frac{{}_l\rho_0\, {}_lCp_0 \Delta x_l T_0}{2\Delta t} + \dot{q}_{\text{net}}^* = 0 \tag{5}$$

Thus, Eq. (3) may be rewritten in terms of known and unknown quantities as follows:

$$h = (1/a_0)(f - f^*) - b_0(T_0' - T_0'^*) - (\dot{q}_{\text{net}} - \dot{q}_{\text{net}}^*) = 0 \tag{6}$$

Since both f and f^* are zero by Eqs. (4) and (5), this results in

$$-b_0(T_0' - T_0'^*) - (\dot{q}_{\text{net}} - \dot{q}_{\text{net}}^*) = 0 \tag{7}$$

and the desired correction from Eq. (7) is

$$\dot{q}_{\text{net}}^* = \dot{q}_{\text{net}} + b_0(T_0' - T_0'^*) = \dot{q}_{\text{net}} + b_0 \Delta T_0' \tag{8}$$

($\Delta T_0'$ may be approximated by the temperature difference between the predicted temperature and measured temperature response at the thermocouple location because of the next step in the algorithm.) The initial value of $\dot{q}_{\text{net}}$ usually is chosen to be the converged value for the previous time step. For the first time step, an arbitrary value such as 1 may be used.

Because the thermocouples generally are located internally rather than at the surface, application of the surface energy balance correction will result in a monotonic approach to the desired solution, but convergence is very slow and may require an excessive number of iterations. Therefore, as a practical solution, it is necessary to switch to an alternate technique. The method of false position can be used to obtain the next estimate of the solution from

$$y_{i+1} = (x_{i-1} y_i - x_i y_{i-1}) / (x_{i-1} - x_i) \tag{9}$$

where $y_i = (\dot{q}_{\text{net}})_i$, $x_i = [T_r' - T_n^*(t)]_i$, T_r' is the calculated value of the temperature at node r corresponding to the thermocouple location, and $T_n^*(t)$ is the temperature given by the thermocouple at that time.

The iterative process is halted when

$$|T_r' - T_n^*(t)| \leq \delta |T_n^*(t)| \tag{10}$$

where δ is the relative convergence tolerance.

If convergence has not been achieved after using the reguli falsi approximation, it can be obtained by using the quadratic fit. Assuming three points, $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, and $P_3(x_3,y_3)$, a quadratic equation can be formed to include all three points. The equation is

$$y = Ax^2 + Bx + C \tag{11}$$

where A, B, and C are quadratic coefficients. By substituting each of the three points into Eq. (11), one has a system with three equations and three unknowns (the coefficients A, B, and C). After the coefficients have been determined, a point on the quadratic curve may be found. The points have been formed such that solution is at $y = C$ or at the point $P_4(0, C)$.

If, in evaluating the original function with C to obtain a new x_4, the solution is not within the desired tolerance as required by Eq. (10) (i.e., $|x_4| < \epsilon$), P_4 is substituted for one of the previous points (e.g., one with the largest $|x|$), and the coefficients are determined again. The process is repeated until the desired solution is obtained. Normally, this process requires only one quadratic iteration since the surface energy balance correction and reguli falsi techniques are converging to the desired solution.

Numerical difficulties arise in determining the surface heat rate or the surface temperature from data based on interior thermocouples. This difficulty is partly due to the timelag imposed on the system resulting from the finite distance between the surface and the thermocouple location. Another factor is the damping of surface changes at the thermocouple

Table 1 Thermophysical properties of RSI (density 144 kg/m³)

Temperature, K	Transverse thermal conductivity, W/(m·K), at a pressure, N/m², of -					Specific heat J/(kg·K)
	10.13	101.3	1013	10 132	101 325	
117	0.009	0.013	0.026	0.038	0.040	293
172	.010	.014	.029	.040	.043	439
256	.013	.017	.032	.043	.048	628
394	.016	.022	.039	.055	.059	879
533	.022	.029	.048	.069	.075	1054
672	.030	.038	.056	.085	.092	1151
811	.040	.048	.068	.104	.114	1205
950	.053	.060	.085	.125	.135	1238
1089	.072	.079	.107	.151	.163	1256
1200	.092	.100	.127	.176	.189	1264
1228	.098	.105	.133	.183	.196	1268
1339	.121	.130	.156	.212	.226	1268
1367	.127	.135	.163	.219	.235	1268
1422	.140	.148	.174	.235	.252	1268
1450	.147	.156	.182	.244	.261	1268
1533	.167	.177	.200	.268	.288	1268

location. Other errors that may arise are due primarily to the method used for approximating the thermal model, the magnitude of $\dot{q}_{net}$, and the magnitude of the thermocouple temperatures. Of course, it is assumed that time steps compatible with the physical system would be used for the thermal model. It should be noted that both the net heating rate and the surface temperature have unique solutions, whereas the convective heating rate is dependent on the assumed value for emissivity.

Beck's Method

Beck's method[2] differs from the JSC method both in the convergence technique and in the amount of temperature data that may be used to calculate the net heating rate. As with the JSC method, the thermal response of the material is calculated, from surface to backwall, using an assumed heating rate. Beck, however, allows for the use of measured temperatures at times other than the current time. That is, with Beck's method one can use intermediate temperature data at $m-1$ time steps, future temperature data ar $r-1$ time steps, or both.

Beck calculates $m \cdot r$ temperature differences, in a least-squares sense, to calculate a correction to the net heating rate. The iterative process is halted when the change in the net heating rate satisfies a prespecified convergence criterion. This is stated mathematically as

$$|\Delta q_i| \leq \delta |q_{i-1}| \tag{12}$$

where Δq_i is the change in the net heating rate at the ith iteration, q_{i-1} is the net heating rate used at the $(i-1)$th iteration, and δ is the relative convergence criterion.

It should be pointed out that, although the convergence criteria in Eqs. (10) and (12) have the same effect, their relative magnitudes differ. A convergence criterion of 1×10^{-6} in Eq. (10) corresponded to a convergence criterion of 1×10^{-3} in Eq. (12) when comparing analytical test case results.

Efficiency Comparison

There is a significant difference between the computation cost in utilizing the JSC method and in utilizing Beck's method. Beck's method is designed to solve a linear problem in one iteration, whereas the JSC method is designed to solve a nonlinear problem in two iterations. As previously mentioned, the JSC method usually converges in three evaluations of the implicit tridiagonal solution at each time step, and if further refinement is required each additional iteration requires only one additional evaluation. On the other hand, Beck's method requires $4 \cdot m \cdot r$ evaluations of the implicit tridiagonal solution for the first iteration, and $2 \cdot m \cdot r$ evaluations for every iteration thereafter. Thus the two methods require the same number of evaluations if and only if the JSC method converges at the end of three iterations and Beck's method with $m=1$ and $r=1$ can converge in one iteration.

Analytical Verification

The numerical methods discussed in the previous section have been evaluated for typical Space Shuttle Orbiter materials and environments. In general, the Space Shuttle Orbiter reusable TPS consists of reusable surface insulation (RSI) for areas with maximum surface temperature of less than 1533 K (2759.4°R) and a reusable carbon-carbon (RCC) where surface temperature exceeds 1533 K (2759.4°R). The thermophysical properties used in this investigation are presented in Table 1.

The analytical data were obtained by solving an implicit, one-dimensional, thermal model using known boundary conditions. The resulting temperature history from one of the internal nodes then was used by the inverse programs, JSC and Beck methods, as a boundary condition. This simulates the temperature history that would be provided from thermocouple data in an experimental case. The inverse programs, in turn, used this transient data to determine the now unknown surface conditions, the net heating rate, and temperature. This allowed for a direct comparison between the known convective heating rate and that predicted by both Beck's method and the JSC method.

Comparisons were made for two cases. The first case consisted of a linear thermal model (constant thermal properties and a surface emittance of zero) subjected to a triangular heating rate. This case is similar to the one used by Beck[2] and served to verify the correct implementation of his algorithm. The results of this investigation are summarized in Table 2. From Table 2, it can be seen that, for Beck's method

Table 2 Comparison of the average error in determining the heat rate for the linear problem

Method	Iteration Count	Convergence criterion (δ)	Average Error (BTU/ft²/sec)	Average Error (W/m²)
JSC	72	1×10^{-4}	2.405×10^{-3}	2.729×10^{1}
JSC	72	1×10^{-5}	2.405×10^{-3}	2.729×10^{1}
JSC	72	1×10^{-6}	2.405×10^{-3}	2.729×10^{1}
JSC	84	1×10^{-7}	4.642×10^{-4}	5.268×10^{0}
Beck m-1, r=1	98	5×10^{-3}	6.590×10^{-4}	7.479×10^{0}
Beck m=1, r-2	192	5×10^{-3}	1.319×10^{-1}	1.497×10^{3}
Beck m=1, r=3	288	5×10^{-3}	2.256×10^{-1}	2.560×10^{3}
Beck m=2, r=1	---	5×10^{-3}	Unstable	Unstable
Beck m=2, r=2	384	5×10^{-3}	2.802×10^{-1}	3.180×10^{3}
Beck m-2, r=3	576	5×10^{-3}	3.126×10^{-1}	3.548×10^{3}

with $m=1$ and $r=1$, the average error§ for a convergence criterion of 5×10^{-3} on the net heating rate falls somewhere between the errors calculated for the convergence criteria of 10^{-6} and 10^{-7} on temperature using the JSC method. Table 2 shows that with $m=1$ the use of future times did not improve the accuracy. Only when the intermediate times were used ($m=2$) did the accuracy improve with using future times. For $m=2$ and $r=2$, the method became unstable, and none of the results using intermediate times ($m=2$) were as accurate as when no intermediate times ($m=1$) were used. The only methods that were able to handle the abrupt change in curvature in the heating were the $m=1$ and $r=1$ case for Beck's method and the JSC method.

The second case consisted of a nonlinear RSI thermal model (Fig. 1) (temperature- and pressure-dependent thermal properties and a nonzero surface emittance) subjected to heating rate history typical of a Shuttle Space Orbiter entry. The results obtained using both the JSC and Beck methods were similar to those obtained in the linear case. The results for both the linear and nonlinear models indicate that for analytical data no advantage is gained by using future times.

Another objective in this analytical investigation was to determine the effects of thermocouple depth x_j and the convergence criterion δ on the accuracy in the calculation of the heating rate. Using the JSC method, the range of values for δ was 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, with thermocouple depths of 0.254, 0.508, 0.762, and 1.016 cm (0.1, 0.2, 0.3, and 0.4 in.) from the heated surface. The effects of the convergence criterion and thermocouple depth on the average error can be seen in Table 3. Basically the results indicate that for each additional 0.254-cm (0.1 in.) increase in depth of the thermocouple the convergence criterion must be decreased by a factor of 10 to maintain the same relative accuracy. [It should be noted that a thermocouple depth of 0.254 cm (0.1 in.) corresponds to a dimensionless time step¶ of 0.4, whereas a thermocouple depth of 1.016 cm (0.4 in.) corresponds to a dimensionless time step of 0.02.]

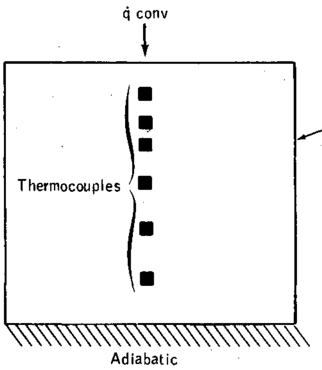

Fig. 1 RSI thermal model.

Table 3 Comparison of the average error in determining the heat rate for the RSI trajectory at different thermocouple depths using the JSC method

Lead thermocouple depth (cm)	(inches)	Convergence criterion (δ)	Average Error (BTU/ft^2-sec)	(W/m^2)
0.254	0.1	1×10^{-4}	1.372×10^{-2}	1.557×10^{2}
		1×10^{-5}	6.954×10^{-4}	7.892×10^{0}
		1×10^{-6}	2.587×10^{-4}	2.936×10^{0}
		1×10^{-7}	2.587×10^{-4}	2.936×10^{0}
0.508	0.2	1×10^{-4}	5.502×10^{-2}	6.244×10^{2}
		1×10^{-5}	7.228×10^{-4}	8.203×10^{0}
		1×10^{-6}	2.823×10^{-4}	3.204×10^{0}
		1×10^{-7}	2.564×10^{-4}	2.910×10^{0}
0.762	0.3	1×10^{-4}	3.484×10^{-1}	3.954×10^{3}
		1×10^{-5}	1.752×10^{-2}	1.988×10^{2}
		1×10^{-6}	2.053×10^{-3}	2.330×10^{1}
		1×10^{-7}	4.107×10^{-4}	4.661×10^{0}
1.016	0.4	1×10^{-4}	1.442×10^{0}	1.637×10^{4}
		1×10^{-5}	1.396×10^{-1}	1.584×10^{3}
		1×10^{-6}	2.354×10^{-2}	2.672×10^{2}
		1×10^{-7}	1.523×10^{-3}	1.728×10^{1}

§The average error equals

$$\left[\frac{1}{n}\sum_{i=1}^{n}(\dot{q}_i-\dot{q}_i^*)^2\right]^{1/2}$$

where $\dot{q}_i$ is the calculated convective heating rate, $\dot{q}_i^*$ is the actual convective heating rate, and n is the total number of individual measurements i taken.

Experimental Data

Thermal evaluation tests have been conducted in the NASA/JSC Radiant Heat Test Facility on test models fabricated from RSI and covered with a high-emittance surface wash. The test specimen consisted of a set of staggered 33.02×33.02-cm (13-×13-in.) tiles 5.08 cm (2 in.) thick bonded to a SIP (strain isolation pad), which, in turn, was bonded to an aluminum plate attached to T-bars. There was no spacing between the tiles, i.e., a no-gap configuration. The test environment was designed to simulate entry heating and pressure conditions.

Thermal Model

The thermal model consisted of 5.08 cm (2 in.) of RSI subdivided into approximately 20 nodes. This subdivision was accomplished such that node centers were forced at thermocouple locations. The boundary conditions were heating at the surface and adiabatic at the backwall.** The wash on the surface was ignored, since both its thermal properties and depth of penetration were unknown.†† The initial temperature profile on the interior nodes was determined from a linear interpolation of the initial interior thermocouple test data. It was assumed that the temperature from the last thermocouple to the backwall remained constant. The initial surface temperature and intervening node temperatures were obtained by linearly extrapolating the temperature from the first two interior thermocouples.

Test Procedure and Identification

Thermocouple plugs were installed in the center of the end tile in each quadrant and in the center of the T-bar panel [approximately 127×152.4 cm (50×60 in.)] (see Fig. 2). Each thermocouple plug was 3.81 cm (1.5 in.) in diameter with five thermocouples. The thermocouples were located at 0.0, 0.381, 1.27, 2.286, and 3.81 cm (0.0, 0.15, 0.5, 0.9, and 1.5 in.) from the heated surface.‡‡

¶The dimensionless time step is given by $\Delta\tau=\alpha\Delta t/E^2$, where $\Delta\tau$ is the dimensionless time step, α is the thermal diffusivity, Δt is the time step used in the thermal model, and E is the thermocouple depth. Instability in the JSC iterative techniques usually occurs with $\Delta\tau>0.018$.

**The use of the adiabatic boundary condition for this test environment and specimen has been justified by separate analysis. This analysis compared the surface temperatures predicted by using a measured temperature history boundary condition at the backwall to those predicted by using an adiabatic boundary condition. Temperature differences were not observed until cooldown had been initiated, where the model using adiabatic boundary condition underpredicted the model using a temperature history boundary condition. The maximum temperature difference was less than 11 K (20°R).

††The surface wash was used to provide an emittance of approximately 0.9. It is assumed that the penetration is not deep and that the surface temperature is more dependent on the emittance of the surface than any material property characteristics exhibited by the wash.

‡‡The lead thermocouple actually was embedded slightly into the material and was below the surface. Its distance from the surface is unknown.

The T-bar panel was placed in the chamber with the RSI surface directed toward the radiant lamps. The lamps were graphite heater rods encapsulated in a nitrogen environment. Heating was controlled by monitoring a set of 13 control thermocouples lying midway between the thermocouple plugs in the first and fourth quadrants and the center thermocouple plug. During the test, the chamber pressure data and thermocouple data were recorded at 1-sec intervals on magnetic tape. This magnetic tape constitutes the main data base used in this report to verify the numerical method. The data that appear on the magnetic tape are generally in too rough of a form to be used successfully for determining the surface conditions. To smooth the data, recourse is made to a least-squares procedure utilizing orthogonal polynomials. The polynomial is fit over successively overlapping data sets to obtain a continuous temperature.

Results

Only two of the five thermocouple plugs shown in Fig. 2 are presented, since similar results were obtained for other test specimens. The two embedded thermocouples that provided the data used for temperature prediction are labeled 1R14 and 3R20. The corresponding surface thermocouples to which the predictions were compared were 1R13 and 3R19.

The surface temperatures predicted by the JSC method were based on smoothed data, whereas the surface temperatures predicted by Beck's method were based on the "raw" unsmoothed data using one ($r=2$) and two ($r=3$) future times. The surface tempeature predictions by the JSC method and Beck's method (one future time) for thermocouple 1R14 can be seen in Fig. 3. During the first few steps, the JSC method oscillated, and then the surface temperature predictions became smooth, whereas with Beck's method the initial steps were smooth with oscillations occurring later throughout the data. When using two future times with Beck's method, these oscillations were still present (see Fig. 4), but the magnitude was reduced. The oscillations observed with Beck's method can be attributed to using raw-unsmoothed data. For thermocouple 3R20, no oscillations were observed for the JSC method. Beck's method using one future time (Fig. 5) displayed oscillations at several times in the surface temperature predictions. These oscillations were

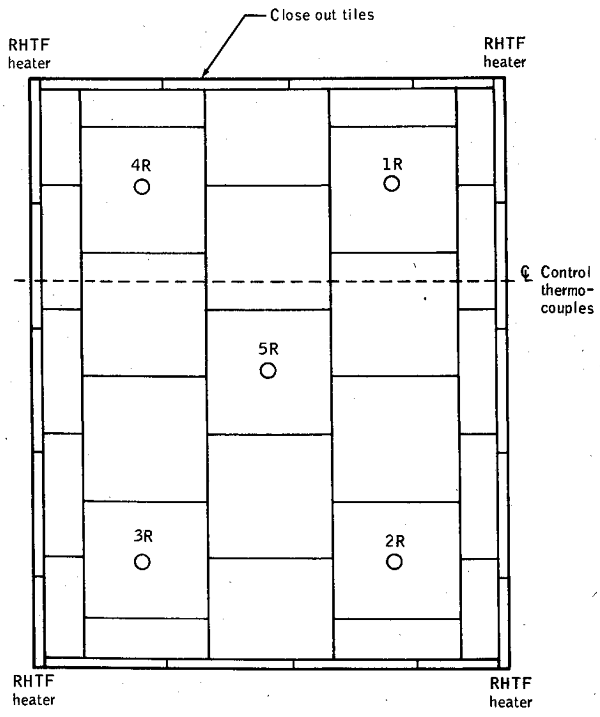

Fig. 2 TPS radiant heatsink test.

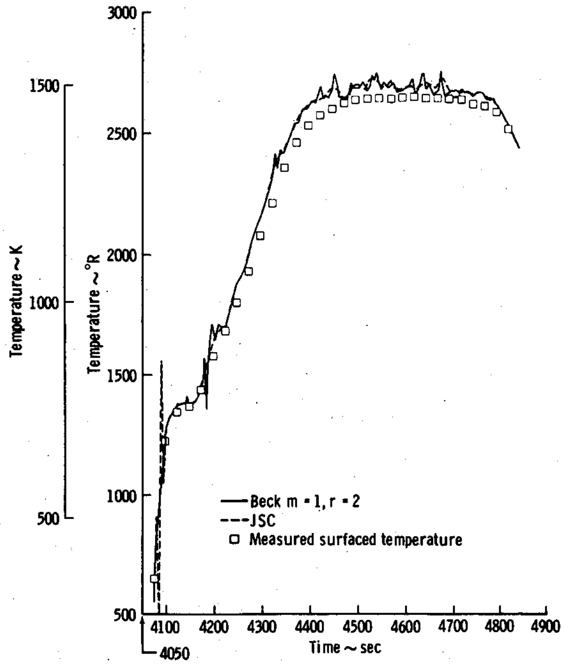

Fig. 3 Predicted values of surface temperature using the JSC method and Beck's method (one future time) from thermocouple 1R14.

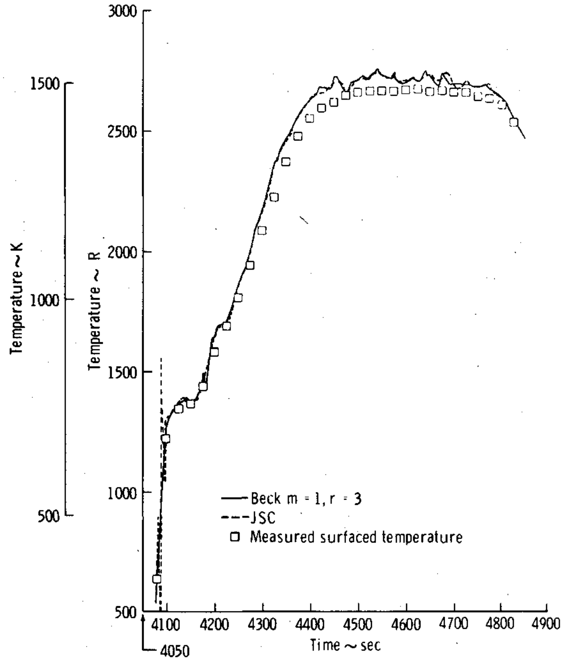

Fig. 4 Predicted values of surface temperature using the JSC method and Beck's method (two future times) from thermocouple 1R14.

dampened considerably when two future times were used (see Fig. 6).

In comparing the surface temperatures to those measured on the surface, it was observed that the predicted temperatures were consistently higher than those measured. This overprediction is attributed to the embedding of the lead thermocouple.

Overall the agreement between the JSC method and Beck's method was good, with major discrepancies only when oscillations were observed. With the JSC method, the only oscillations observed were during the initial start-up phase. Beck's method did not display any oscillations during this phase, but oscillations did occur later in the data for all tests. This was due to the thermocouple recording errors that are always present. As was mentioned, the computational cost in

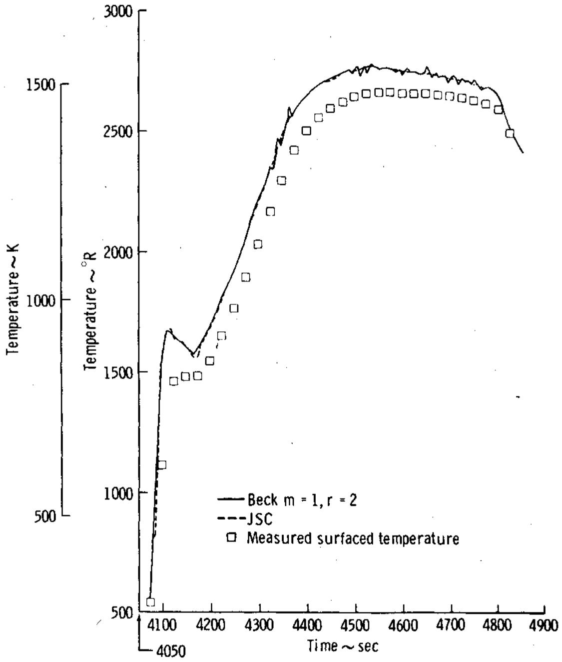

Fig. 5 Predicted values of surface temperatures using the JSC method and Beck's method (one future time) from thermocouple 3R20.

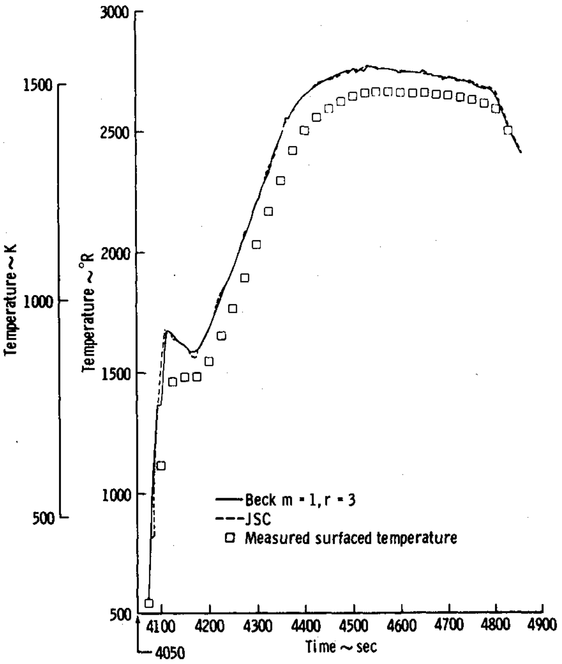

Fig. 6 Predicted values of surface temperature using the JSC method and Beck's method (two future times) from thermocouple 3R20.

using the JSC method is much smaller than the cost of using Beck's method with future times (see Table 4). Other than the problem with the initial oscillation that occurred with plug 1R, there was no advantage in using future times. It should be mentioned that, if the data were noisy at the initial time, Beck's method would have oscillated there also.

Concluding Remarks

An inverse solution technique using a single embedded thermocouple has been developed for predicting the transient thermal environment to which the Space Shuttle Orbiter thermal protection system is exposed during entry. The accuracy of the numerical method has been demonstrated for a low-conductivity material by comparison with experimental and analytical data.

A comparison also was made between the method developed by Beck and the method developed at JSC for solving the inverse problem using analytical and experimental data. The results of this investigation indicated that no advantage will be gained by using Beck's method with future temperatures.

The procedure developed is quite general and has been incorporated into a previously developed program used to compute thermal conductivity values from experimental data. Thus, a capability now exists for computing surface conditions (heat flux and/or temperature) and thermal conductivity values using the data from a single experiment.

Table 4 Comparison of the iteration count in determining the heat rate for the experimental trajectory using thermocouple 1R14

Time (sec)	JSC[a]	Beck[a] m=1, r=1	Method Beck[b] m=1, r=2	Beck[b] m=1, r=3
4075	0	0	0	0
4100	15	20	44	66
4200	75	98	228	318
4300	135	174	392	546
4400	195	250	548	786
4500	255	328	708	1026
4600	314	404	860	1266
4700	374	478	1020	1506
4800	432	554	1180	1734
4900	492	640	1340	1974

[a] Used smoothed data. [b] Used raw data.

References

[1] Curry, D.M. and Williams, S.D., "Nonlinear Least Squares—An Aid to Thermal Property Determination," *AIAA Journal*, Vol. 11, May 1973, pp. 670-674.

[2] Beck, J.V., "Nonlinear Estimation Applied to the Nonlinear Inverse Heat Conduction Problem," *International Journal of Heat and Mass Transfer*, Vol. 13, April 1970, pp. 703-716.

[3] Beck, J.V. and Wolf, H., "The Nonlinear Inverse Heat Conduction Problem," American Society of Mechanical Engineers, Paper 65-HT-40, 1965.

[4] Stolz, G., Jr., "Numerical Solutions to an Inverse Problem of Heat Conduction for Simple Shapes," *Journal of Heat Transfer*, Vol. 82, Ser. C, Feb. 1960, pp. 20-26.

[5] Frank, I., "An Application of Least Squares Method to the Solution of the Inverse Problem of Heat Conduction," *Journal of Heat Transfer*, Vol. 85, Ser. C, Nov. 1963, pp. 378-379.

[6] Burggraf, O.R., "An Exact Solution of the Inverse Problem in Heat Conduction Theory and Applications," *Journal of Heat Transfer*, Vol. 85, Ser. C, Aug. 1964, pp. 373-382.

[7] Sparrow, E.M., Haji-Sheikh, A., and Lundgren, T.S., "The Inverse Problem in Transient Heat Conduction," *Journal of Applied Mechanics*, Vol. 31, Ser. E., Sept. 1964, pp. 369-375.

[8] Powell, W.B. and Price, T.W., "A Method for the Determination of Local Heat Flux from Traisient Temperature Measurements," *ISA Transactions*, Vol. 3, March 1964, pp. 246-254.

[9] Howard, F.G., "Single-Thermocouple Method for Determining Heat Flux to a Thermally Thick Wall," NASA TN D-4737, 1968.

[10] Cornette, E.S., "Forebody Temperatures and Total Heating Rates Measured During Project Fire 1 Reentry at 38,000 Feet Per Second," NASA TM X-1120, 1965.

[11] Cornette, E.S., "Forebody Temperatures and Calorimeter Heating Rates Measured During Project Fire 2 Reentry at 11.35 Kilometers Per Second," NASA TM X-1305, 1966.

[12] Imber, M. and Khan, J., "Prediction of Transient Temperature Distributions With Embedded Thermocouples," *AIAA Journal*, Vol. 10, June 1972, pp. 784-789.

[13] Christensen, H.E. and Kipp, H.W., "Heating in Shuttle RSI Gaps Derived from an Inverse Heat Transfer Solution," *Fifth Intersociety Conference on Environmental Systems*, American Society of Mechanical Engineers, Paper 75-ENAS-15, July 1975.

[14] Kunz, K.S., *Numerical Analysis*, McGraw-Hill, New York, 1957, pp. 4-6.